www.sciencevision.org

Original Research

Sci Vis Vol 13 Issue No 3
July-September 2013
ISSN (print) 0975-6175
ISSN (online) 2229-6026

Phase velocity of harmonic waves in monoclinic anisotropic medium

S. S. Singh[1] and L. Krosspanie

Department of Mathematics & Computer Science, Mizoram University, Aizawl 796 004, Mizoram, India

Received 13 June 2013 | Revised 8 August 2013 | Accepted 22 September 2013

Abstract

The problem of phase velocity of plane waves in monoclinic anisotropic medium was investigated. The phase velocity of Quasi-nature elastic waves was obtained analytically and numerically for a particular model. It was observed that the phase velocity of elastic waves is functions of the angle of propagation, elastic constants and density of the medium. These phase velocities were computed numerically for a particular model.

Key words: Phase velocity; monoclinic medium; qP, qSV & qSH-waves.

Introduction

The problem of elastic wave propagation in different media is an important phenomenon in the field of seismology, earthquake engineering and geophysics. The elastic waves propagating through the Earth (called seismic waves) have to travel through different layers and interfaces. These waves have different velocities and are influenced by the properties of the layer through which they travel. The signals of these waves are not only helpful in providing information about the internal structures of the Earth but also helpful in exploration of the valuable materials such as minerals, crystals and metals etc. This technique is one of the most suitable in terms of time saving and economy. Nayfeh discussed the analytical expressions for the reflection and refraction coefficients from the interfaces of liquid-anisotropic half-spaces possessing up to as low as monoclinic symmetry and the expressions for the distributions of stresses and displacements throughout the fluid-solid system.[1] Chattopadhyay and Choudhury studied the problem of reflection of *P*-waves at plane boundary of a half-space of monoclinic type.[2] Later, Chattopadhyay and Saha investigated the reflection and refraction of P-waves at a plane interface between two different monoclinic media.[3] Chattopadhyay *et al.* discussed the problem of the reflection and refraction of shear waves in monoclinic media.[4] Chattopadhyay and Saha investigated the problem of the reflection and

Corresponding author: Singh
Phone: +91-09863274645
E-mail: saratcha32@yahoo.co.uk

refraction of quasi-SV-waves at the interface of the monoclinic media.[5] Later, Singh and Khurana studied the problem of reflection of P- and SV-waves at a free surface of a monoclinic elastic half space.[6] Singh and Khurana also studied the problem of the reflection and refraction of P- and SV-waves at the interface between two monoclinic elastic half-spaces.[7] Gupta attempted the problem of the reflection and transmission of SH-waves in laterally and vertically heterogeneous media at an irregular boundary.[8] Kaur and Tomar discussed the problem of reflection and refraction of SH-waves at a corrugated interface between two monoclinic elastic half-spaces.[9] Singh and Tomar attempted the problem of reflection and refraction of elastic waves for the incident qP-wave at a corrugated interface between two dissimilar monoclinic half-spaces.[10] They derived the reflection and refraction coefficients of the reflected and refracted waves.

In this article, the problem of phase velocity of the plane harmonic waves in the monoclinic anisotropic medium has been investigated. The phase velocity of the qP-wave, qSV-wave and qSH-wave are derived analytically and numerically. It has been observed that these phase velocities are found to be the functions of angle of incidence, elastic parameters and density of the medium.

Field Equations

The constitutive equations in homogeneous anisotropic elastic material of monoclinic type with the x_2x_3-plane as the plane of symmetry, are given by

$$\begin{aligned}
\tau_{11} &= c_{11}e_{11} + c_{12}e_{22} + c_{13}e_{33} + 2c_{14}e_{23},\\
\tau_{23} &= c_{14}e_{11} + c_{24}e_{22} + c_{34}e_{33} + 2c_{44}e_{23},\\
\tau_{22} &= c_{12}e_{11} + c_{22}e_{22} + c_{23}e_{33} + 2c_{24}e_{23},\\
\tau_{33} &= c_{13}e_{11} + c_{23}e_{22} + c_{33}e_{33} + 2c_{34}e_{23},\\
\tau_{13} &= 2(c_{55}e_{13} + c_{56}e_{12}),\\
\tau_{12} &= 2(c_{56}e_{13} + c_{66}e_{12}), \qquad (1)
\end{aligned}$$

where $2e_{ij} = u_{i,j} + u_{j,i}$; u_i are components of displacement vector $\mathbf{u}(\mathbf{x_2}, \mathbf{x_3}, \mathbf{t})$, τ_{ij} are components of stress tensors, c_{ij} $(i, j = 1, 2, 3, \ldots, 6)$ are elastic constants.

The equations of motion in the absence of body forces are given by

$$\frac{\partial \tau_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}; \qquad (\mathrm{i}, \mathrm{j} = 1, 2, 3), \qquad (2)$$

where ρ is the density of the medium.

Using the stress tractions given by Eq. (1), the equation of motion in (2) reduce to

$$c_{66}\frac{\partial^2 u_1}{\partial x_2^2} + 2c_{56}\frac{\partial^2 u_1}{\partial x_2 \partial x_3} + c_{55}\frac{\partial^2 u_1}{\partial x_3^2} = \rho \frac{\partial^2 u_1}{\partial t^2}, \qquad (3)$$

$$c_{22}\frac{\partial^2 u_2}{\partial x_2^2} + c_{44}\frac{\partial^2 u_2}{\partial x_3^2} + c_{24}\frac{\partial^2 u_3}{\partial x_2^2} + c_{34}\frac{\partial^2 u_3}{\partial x_3^2} + 2c_{24}\frac{\partial^2 u_2}{\partial x_2 \partial x_3} + (c_{23} + c_{44})\frac{\partial^2 u_3}{\partial x_2 \partial x_3} = \rho \frac{\partial^2 u_2}{\partial t^2}, \qquad (4)$$

$$c_{24}\frac{\partial^2 u_2}{\partial x_2^2} + c_{34}\frac{\partial^2 u_2}{\partial x_3^2} + c_{44}\frac{\partial^2 u_3}{\partial x_2^2} + c_{33}\frac{\partial^2 u_3}{\partial x_3^2} + 2c_{34}\frac{\partial^2 u_3}{\partial x_2 \partial x_3} + (c_{23} + c_{44})\frac{\partial^2 u_2}{\partial x_2 \partial x_3} = \rho \frac{\partial^2 u_3}{\partial t^2}. \qquad (5)$$

We note that Eq. (3) is uncoupled in u_1, while Eqs. (4) and (5) are couple in u_2, and u_3. The first equation represents the equation of motion for qSH–waves in the monoclinic anisotropic medium, while last two equations represent the equations of motion for qP and qSV– waves.

Suppose c represents the phase velocity and k represents the wavenumber of plane wave propagating in x_2x_3 plane in the direction having unit propagation vector as $\boldsymbol{p} = (0, p_2, p_3)$. Let us take $\boldsymbol{u} = \{u_1, u_2, u_3\}$ such that

$$\{u_1, u_2, u_3\}(x_2, x_3, t) = A\{d_1, d_2, d_3\}\exp\{ik(ct - p_2x_2 - p_3x_3)\}, \qquad (6)$$

where $\boldsymbol{d} = (d_1, d_2, d_3)$ is the unit displacement vector which also known as the polarization vector.

Using the expressions of u_1, u_2 and u_3 from Eq.(6) into the equation of motion (3)-(5), we have

$$(U - \rho c^2)d_2 + Vd_3 = 0, \qquad (7)$$

$$Vd_2 + (Z - \rho c^2)d_3 = 0, \qquad (8)$$

$$c_{66}p_2^{\ 2} + 2c_{56}p_2p_3 + c_{55}p_3^2 = \rho c^2 \equiv \rho c_2^2, \quad (9)$$

where

$U(p_2, p_3) = c_{22}p_2^2 + c_{44}p_3^2 + 2c_{24}p_2p_3,$
$V(p_2, p_3) = c_{24}p_2^2 + c_{34}p_3^2 + (c_{23} + c_{44})p_2p_3,$
$Z(p_2, p_3) = c_{44}p_2^2 + c_{33}p_3^2 + 2c_{34}p_2p_3.$

With the help of Eqs. (7) and (8), we obtain

$$d_2/d_3 = V/(\rho c^2 - u) = (\rho c^2 - Z)/V \quad (10)$$

This result gives

$$2\rho c^2 \equiv 2\rho c_{1,2}^2 = U + Z \pm \sqrt{(U - Z)^2 + 4V^2} \quad (11)$$

In Eq. (11), 'plus' sign gives the phase velocity of qP-waves, while 'minus' sign gives the phase velocity of qSV-waves and Eq.(9) gives the phase velocity of the qSH-waves. Thus, we have observed that the phase velocity of elastic waves in the monoclinic anisotropic medium depend on angle of propagation, elastic parameters and density of the medium.

Numerical Computations and Results

For the numerical computation of the phase velocities corresponding to the qSV- wave, qP wave and qSH-wave, the following relevant parameters are taken

$c_{24} = 1.1 \times 10^{10}$ N/m², $c_{34} = 8.0 \times 10^{10}$ N/m², $c_{23} = 1.0 \times 10^{9}$ N/m², $c_{33} = 2.02 \times 10^{11}$ N/m², $c_{22} = 2.33 \times 10^{11}$ N/m², $c_{44} = 6.97 \times 10^{11}$ N/m², $c_{66} = 4.50 \times 10^{10}$ N/m², $c_{56} = 7.0 \times 10^{10}$ N/m² & $c_{34} = 8.1 \times 10^{10}$ N/m².

The variations of phase velocity with angle of incidence for different values of density are depicted through Figures 1-3.

In all the figures,

Curve I: $\rho = 2.6 \times 10^3 Kg/m^3$;
Curve II: $\rho = 4.8 \times 10^3 Kg/m^3$;
Curve III: $\rho = 6.6 \times 10^3 Kg/m^3$.

In Figure 1, we have seen that the phase velocity, c_2 corresponding to qSV-wave starts from certain value at the normal angle of incidence which decreases with the increase of angle of incidence and obtained the minimum value at $\theta = 40^0$ which increases thereafter with the increase in θ. With the increase of the density of the medium, the phase velocity is also decreased. In Figures 2 & 3, the phase velocities corresponding to qP and qSH-wave increase with the increase of the angle of incidence and attain the maximum values at certain values of the angle of incidence and then, they decrease with the increase of θ. Similarly as in the above figure, the phase velocities corresponding to the qP and qSH-waves decrease with the increase of the density.

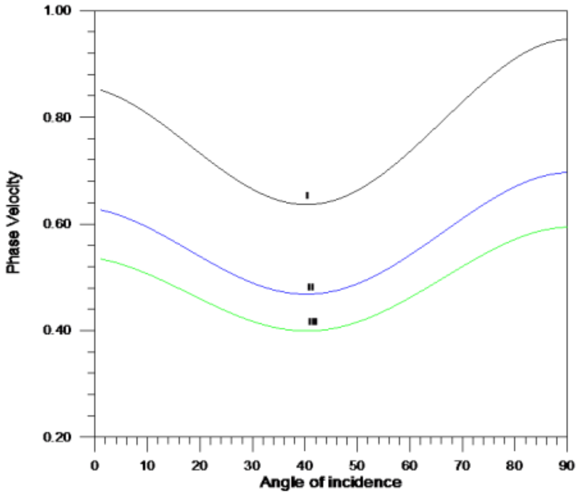

Figure 1. Variation of the modulus of phase velocity of qSV-wave. [Curve I: $\rho = 2.6$; Curve II: $\rho = 4.8$; Curve III: $\rho = 6.6$]

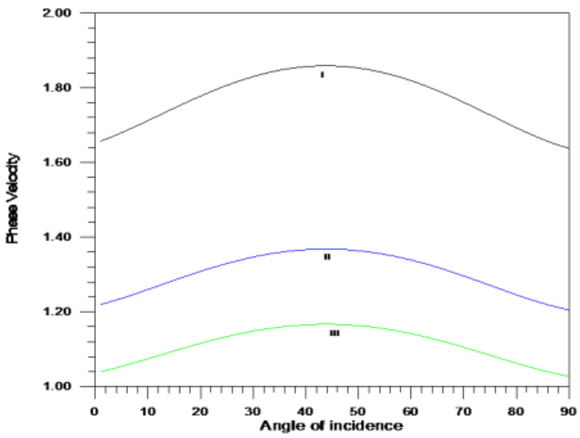

Figure 2. Variation of the modulus of phase velocity of qP-wave. [Curve I: $\rho = 2.6$; Curve II: $\rho = 4.8$; Curve III: $\rho = 6.6$]

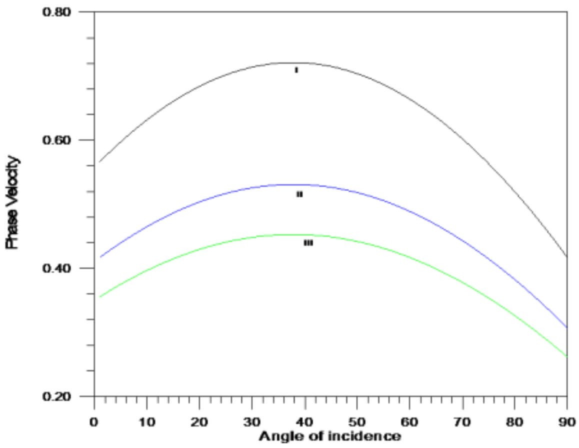

Figure 3. Variation of the modulus of phase velocity of qSH-wave. [Curve I: ρ = 2.6; Curve II: ρ = 4.8; Curve III: ρ = 6.6]

Conclusion

We have derived the phase velocities of the plane wave in the monoclinic medium. The phase velocities corresponding to the qP-wave, qSV-wave and qSH-wave are expressed in the closed form and these velocities are computed numerically for a particular model for different values of the density of the medium. We have seen that with the increase of the density of the medium, all the values of the phase speed corresponding to the qP-wave, qSV-wave and qSH-wave decrease. Thus, we may conclude that the phase velocities are inversely proportional with the density in such medium.

References

1. Nayfeh AH (1991). Elastic wave reflection from liquid–anisotropic substrate interfaces. *Wave Motion,* **14**, 55-67.
2. Chattopadhyay A & Choudhury S (1995). The reflection phenomena of P-waves in a medium of monoclinic type. *Int J Engng Sci*, **33**, 195-207.
3. Chattopadhyay A & Saha S (1999). Reflection and refraction of quasi-SV waves at the interface of the monoclinic media. *Acta Geophys Pol*, **47**, 307-322. ADDENDUM in *Acta Geophys. Pol*, **51**, 473-475 (2003).
4. Chattopadhyay A, Saha S & Chakraborty M (1997). Reflection and transmission of shear waves in monoclinic media. *Int J Numer Anal Meth Geomech*, **21**, 495-504.
5. Chattopadhyay A & Saha S (1996). Reflection and refraction of P-waves at the interface of two monoclinic media. *Int J Engng Sci*, **34**, 1271-1284.
6. Singh SJ & Khurana S (2001). Reflection and transmission of P- and SV-waves at the interface two between monoclinic elastic half-spaces. *Proc Nat Acad Sci India*, **71A**, 305-319.
7. Singh SJ & Khurana S (2002). Reflection of P- and SV-waves at the free surface of a monoclinic elastic half-space. *Proc Indian Acad Sci (Earth Planet Sci)*, **111**, 401-412.
8. Gupta S (1987). Reflection and transmission of SH-waves in laterally and vertically heterogeneous media at an irregular boundary. *Geophys Trans*, **33(2)**, 89-111.
9. Kaur J & Tomar SK (2004). Reflection and refraction of SH-waves at a corrugated interface between two monoclinic elastic half-spaces. *Int J Numer Anal Meth Geomech*, **28**, 1543-1575.
10. Singh SS & Tomar SK (2007). Quasi-P-waves at a corrugated interface between two dissimilar monoclinic elastic half-spaces. *Int J Solids Struct*, **44**, 197-228.