Science Vision 15(2)

Original Research

2015
April-June
ISSN (print) 0975-6175
ISSN (online) 2229-6026

Phase velocities of elastic waves in swelling porous materials

R. Lianngenga[1]*, Lalawmpuia[1] and J. Lalvohbika[2]

[1]*Department of Mathematics and Computer Science, Mizoram University, Aizawl 796004, India*
[2]*Department of Mathematics, Pachhunga University College, Aizawl 796001, India*

Received 11 February 2015 | Revised 10 May 2015 | Accepted 21 May 2015

Abstract

The present article deals with the study of phase velocities of elastic waves in the medium of swelling porous. The medium consist of mixtures of solid, fluid and gas. Based on Eringen's linear theory of swelling porous, it is found that the existence of three longitudinal waves and two transverse waves, propagating with distinct velocities which are attenuated. Discussions of the particular cases are presented. Numerical and analytical calculations of phase velocities and attenuation are depicted graphically.

Key words: Attenuation; longitudinal wave; porous; phase velocity; swelling; transverse wave.

Introduction

The propagations of waves in swelling porous medium is a subject of continued interest due to its wide and far application in various field of technology, engineering, oil exploration, geophysics, architecture etc. Biological materials, minerals and synthetic porous materials often exhibit the swelling or shrinking when in contact with changing salt concentrations. There is multiplicity of theories which described the mechanical properties of porous materials. Biots introduced one of the earliest theories called Biot's consolidation theory of fluid saturated porous solid.[1] A continuum theory of mixtures are studied extensively by Bowen, he considered a particular volume fraction as constitutive variables.[2] Grag and Nayfeh studied the porous media filled with elastic matrix, water and gas within the context of mixture theory.[3] Tuncay and Corapioglu presented a theory of porous media containing two immiscible Newtonian fluids using volume averaging technique.[4] Gales investigated some theoretical problems concerning waves and vibrations in the context of isothermal linear theory of swelling porous elastic solid with fluid or gas, he proposed only two field equations.[5] Kumar *et al.* studied the waves propagation in swelling porous medium of an infinite extent, with the medium consist of solid and fluid. They also studied the reflection phenomenon using appropriate boundary conditions.[6] Tersa and Bennethum derived a transport equation for swelling porous materials that undergo finite deformations.[7] Gales investigated

Corresponding author: Lianngenga
Phone: +91-9612518027
E-mail: rengsi.9@gmail.com

the spatial behaviour of solutions describing harmonic vibrations of right cylinder in an isothermal linear theory of swelling porous elastic soils.[8] Bofill and Quintanilla studied the problem of anti plane shear deformation of swelling porous elastic soils in case of fluid or gas saturation.[9] In this article, we shall follow Eringen's theory, he developed theory of mixtures for the field of swelling, and proposed continuum theory of swelling porous elastic materials consisting of solid, liquid and gas.[10] Eringen's theory is different from Tuncay and Corapioglu. He assumed that the transverse wave of porous medium provided not only solid matrix but also the viscous nature of fluid.

Basic Equations

Let us consider the region B in a physical three-dimensional space, occupied by swelling porous elastic material. Let us assume that this material consist of a mixture of solid, fluid (viscous) and gas (non-viscous). The Field equations for isothermal, linear and homogeneous isotropic swelling porous material is given by (see Eringen[10]):

$$\left.\begin{aligned} t^s_{i,j} + \rho^s f^s_i + p^f_i + p^g_i &= \rho^s \ddot{u}^s_i \\ t^f_{i,j} + \rho^f f^f_i - p^f_i &= \rho^f \ddot{u}^f_i \\ t^g_{i,j} + \rho^g f^g_i - p^g_i &= \rho^g \ddot{u}^g_i \end{aligned}\right\} \quad (1)$$

where,

$t^k_{i,j}$ − *Component of stress Tensor*; u^k − *Component of displacement vector*

f^s_i − *Component of body force*; p^f_i − *Internal body force*

ρ^k − *Mass density*; $k = s, f, g$

The superpose dot denote partial derivative with respect to time and the subscript preceded by coma denote partial derivative with respect to their corresponding coordinate axes. And the Constitutive equations for the given medium are given by Eringen[10] as follows:

$$\left.\begin{aligned} t^s_{ij} &= 2\mu e^s_{ij} + \left(\lambda e^s_{rr} - A^f e^f_{rr} - A^g e^g_{rr}\right)\delta_{ij} \\ t^f_{ij} &= 2\alpha \dot{e}^f_{ij} + \begin{pmatrix}\beta \dot{e}^f_{rr} - A^f e^s_{rr} - A^{ff} e^f_{rr} \\ - A_{fg} e^g_{rr}\end{pmatrix}\delta_{ij} \\ t^g_{ij} &= -\left(A^g e^s_{rr} + A^{gf} e^f_{rr} + A^{gg} e^g_{rr}\right)\delta_{ij} \\ p^f_i &= B^{ff}\left(\dot{u}^f_i - u^s_i\right) + B^{fg}\left(\dot{u}^g_i - u^s_i\right) \\ p^g_i &= B^{fg}\left(\dot{u}^g_i - u^s_i\right) + B^{gg}\left(\dot{u}^g_i - u^s_i\right) \end{aligned}\right\} \quad (2)$$

Where, λ, μ are Lame's parameters or elastic constants; A^f, B^f, B^{ff}, B^{fg}, B^{gg}, A^{ff}, A^{fg}, A^{gg}, α, β are constitutive constants with the properties of $A^{fg} = A^{gf}$; $B^{fg} = B^{gf}$, and δ_{ij} is a Kronecker delta and e^k_{ij} is strain tensor such that

$$e^k_{ij} = \frac{1}{2}\left(u^k_{i,j} + u^k_{j,i}\right) \quad (3)$$

Putting equations (2) and (3) in equation (1) we get the following equations in vector form:

$$\left.\begin{aligned} &(\lambda + \mu) grad\, div\, u^s + \mu\, div\, grad\, u^s - \left(B^{ff} + B^{fg}\right)\dot{u}^s \\ &+ \left(B^{ff} + B^{fg}\right)\dot{u}^f - A^f\, grad\, div\, u^f - \left(B^{fg} + B^{ff}\right)\dot{u}^s \\ &- A^g\, grad\, div\, u^g + \left(B^{gg} + B^{fg}\right)\dot{u}^g = \rho^s \ddot{u}^s \\ &-A^f\, grad\, div\, u^s + B^{fg}\dot{u}^s + B^{ff}\dot{u}^s - A^{ff}\, grad\, div\, u^f \\ &- B^{ff}\dot{u}^f + (\alpha + \beta) grad\, div\, \dot{u}^f + \beta\, div\, grad\, \dot{u}^f \\ &- A^{fg}\, grad\, div\, u^g - B^{fg}\dot{u}^g = \rho^f \ddot{u}^f \\ &-A^g\, grad\, div\, u^s + B^{gg}\dot{u}^s + B^{gf}\dot{u}^s - A^{fg}\, grad\, div\, u^f \\ &- B^{fg}\dot{u}^f - A^{gg}\, grad\, div\, u^g - B^{gg}\dot{u}^g = \rho^g \ddot{u}^g \end{aligned}\right\} \quad (4)$$

Where, the parameters A^p, A^{pq} are dimensionally equal. Similarly B^p and B^{pq} are also dimensionally equal. α and β are fluid viscous parameters, both has same dimensions.

Wave Propagation

In this section we shall investigate longitudinal and transverse wave by introducing scalar and vector potential (w and v) corresponding to longitudinal and transverse wave respectively. We shall use Helmholtz decomposition theorem as

$$u^k = \nabla w^k + \nabla \times v^k \tag{5}$$

Where, ∇ is Laplacian, $\times$ is cross product and $k = s, f, g$, and $\nabla . v^k = 0$.

Putting equation (5) into equation (4), we shall have

$$\begin{gathered}
\left[\begin{matrix}(\lambda + 2\mu)\nabla^2 \\ -\left(B^{ff} + B^{gg} + 2B^{fg}\right)\frac{\partial}{\partial t} - \rho^s \frac{\partial^2}{\partial t^2}\end{matrix}\right] w^s + \\
\left[-A^f \nabla^2 + \left(B^{fg} + B^{ff}\right)\frac{\partial}{\partial t}\right] w^f \\
+\left[-A^g \nabla^2 + \left(B^{gg} + B^{fg}\right)\frac{\partial}{\partial t}\right] w^g = 0 \\
\left[-A^f \nabla^2 + \left(B^{fg} + B^{ff}\right)\frac{\partial}{\partial t}\right] w^s + \\
\left[(\alpha + 2\beta)\frac{\partial}{\partial t}\nabla^2 - A^{ff}\nabla^2 - B^{ff}\nabla^2 - \rho^f \frac{\partial^2}{\partial t^2}\right] w^f \\
+\left[-A^{fg}\nabla^2 - B^{fg}\frac{\partial}{\partial t}\right] w^g = 0 \\
\left[-A^g \nabla^2 + \left(B^{fg} + B^{gg}\right)\frac{\partial}{\partial t}\right] u^s \\
+\left[-A^{fg}\nabla^2 - B^{fg}\frac{\partial}{\partial t}\right] u^f + \\
\left[-A^{gg}\nabla^2 - B^{gg}\frac{\partial}{\partial t} - \rho^g \frac{\partial^2}{\partial t^2}\right] w^g = 0
\end{gathered} \tag{6}$$

and,

$$\begin{gathered}
\left[\mu\nabla^2 - \left(B^{ff} + B^{gg} + 2B^{fg}\right)\frac{\partial}{\partial t} - \rho^s\right] v^s \\
+\left[\left(B^{ff} + B^{gg}\right)\frac{\partial}{\partial t}\right] v^f \\
+\left[\left(B^{fg} + B^{gg}\right)\frac{\partial}{\partial t}\right] v^g = 0 \\
\left[\left(B^{ff} + B^{gg}\right)\frac{\partial}{\partial t}\right] v^s + \left[\beta\frac{\partial}{\partial t}\nabla^2 - B^{ff}\frac{\partial}{\partial t} - \rho^f\right] v^f \\
+\left[-B^{fg}\frac{\partial}{\partial t}\right] v^g = 0 \\
\left[\left(B^{fg} + B^{gg}\right)\frac{\partial}{\partial t}\right] v^s + \left[-B^{fg}\frac{\partial}{\partial t}\right] v^f \\
+\left[-B^{gg}\frac{\partial}{\partial t} - \rho^g\right] v^g = 0
\end{gathered} \tag{7}$$

corresponding to Longitudinal wave and transverse wave respectively.

For the propagation of time harmonic waves we shall use the following relations

$$\{w^k, v^k\} = \{w_0^k, v_0^k\} e^{ik(n.r - ct)} \tag{8}$$

Where, $i = \sqrt{-1}$, k denote wave number, n is unit normal vector, r is radius vector, c is phase velocity and t is time.

Longitudinal wave

Putting equation (8) into equation (6) we get

$$\left.\begin{aligned}
&(A_1 x^2 + A_2) w_0^s + (B_1 x^2 + B_2) w_0^f \\
&\quad +(C_1 x^2 + C_2) w_0^g = 0 \\
&(D_1 x^2 + D_2) w_0^s + (E_1 x^2 + E_2) w_0^f \\
&\quad +(F_1 x^2 + F_2) w_0^g = 0 \\
&(G_1 x^2 + G_2) w_0^s + (H_1 x^2 + H_2) w_0^f \\
&\quad +(I_1 x^2 + I_2) w_0^g = 0
\end{aligned}\right\} \tag{9}$$

where,

$$A_1 = \frac{\lambda + 2\mu}{\rho^s}; \quad A_2 = \frac{-\left(B^{ff} + B^{gg} + 2B^{fg}\right)i}{\omega\rho^s} - 1$$

$$B_1 = \frac{-A^f}{\rho^s}; \quad B_2 = \frac{\left(B^{fg} + B^{ff}\right)i}{\omega\rho^s}$$

$$C_1 = \frac{-A^g}{\rho^s}; \quad C_2 = \frac{\left(B^{fg} + B^{gg}\right)i}{\omega\rho^s}$$

$$D_1 = \frac{-A^f}{\rho^f}; \quad D_2 = \frac{-\left(B^{fg} + B^{ff}\right)i}{\omega\rho^f}$$

$$E_1 = \left(\frac{\lambda + 2\mu}{\rho^f}\right) i\omega + \frac{A^{ff}}{\rho^f}; \quad E_2 = \frac{B^{ff} i}{\omega\rho^f} + 1$$

$$F_1 = \frac{A^{fg}}{\rho^f}; \quad F_2 = \frac{B^{fg} i}{\omega\rho^s}$$

$$G_1 = \frac{-A^g}{\rho^g}; \quad G_2 = \frac{-\left(B^{fg} + B^{gg}\right)i}{\omega\rho^g}$$

$$H_1 = \frac{A^{fg}}{\rho^g};$$

$$H_2 = \frac{B^{fg} i}{\omega \rho^g}$$

$$I_1 = \frac{A^{gg}}{\rho^g} \quad ;$$

$$I_2 = \frac{B^{gg} i}{\omega \rho^g} + 1$$

and, $x^2 = c^{-2}$.

We can write the equation into the following matrix form:

$$\begin{pmatrix} A_1x^2 + A_2 & B_1x^2 + B_2 & C_1x^2 + C_2 \\ D_1x^2 + D_2 & E_1x^2 + E_2 & F_1x^2 + F_2 \\ G_1x^2 + G_2 & H_1x^2 + H_2 & I_1x^2 + I_2 \end{pmatrix} \begin{pmatrix} u^s \\ u^f \\ u^g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{10}$$

For non-trivial solution of equation (10), we must have

$$det \begin{pmatrix} A_1x^2 + A_2 & B_1x^2 + B_2 & C_1x^2 + C_2 \\ D_1x^2 + D_2 & E_1x^2 + E_2 & F_1x^2 + F_2 \\ G_1x^2 + G_2 & H_1x^2 + H_2 & I_1x^2 + I_2 \end{pmatrix} = 0$$

solving we get

$$Ax^6 + Bx^4 + Cx^2 + D = 0 \tag{11}$$

putting back $x^2 = c^{-2}$, we get

$$Dc^6 + Cc^4 + Bc^2 + A = 0 \tag{12}$$

where, $A = A_1E_1I_1 - A_1F_1H_1 - B_1D_1I_1 + B_1F_1G_1 + C_1D_1H_1 - C_1D_1H_1 - C_1E_1G_1$

$B = A_1E_1I_2 + A_1I_1E_2 - A_1F_1H_2 - A_1H_1F_2 + E_1I_1A_2 - F_1H_1A_2 - B_1D_1I_2 - B_1I_1D_2 + B_1F_1G_2 + B_1G_1F_2 - D_1I_1B_2 + F_1G_1B_2 + C_1D_1H_2 + C_1H_1D_2 - C_1E_1G_2 - C_1G_1E_2 + D_1H_1C_2 - E_1G_1C_2$

$C = A_1E_2I_2 - A_1F_2H_2 + E_1A_2I_2 + I_1A_2E_2 - F_1A_2H_2 - H_1A_2F_2 - B_1D_2I_2 + B_1F_2G_2 - D_1B_2I_2 - I_1B_2D_2 + F_1B_2G_2 + G_1B_2F_2 + C_1D_2H_2 - C_1E_2G_2 + D_1C_2H_2 + H_1C_2D_2 - E_1C_2G_2 - G_1C_2E_2$

$D = A_2E_2I_2 - A_2F_2H_2 - B_2D_2I_2 + B_2F_2G_2 + C_2D_2H_2 - C_2D_2H_2 - C_2E_2G_2$

Equation (12) is cubic in c^2 and the solution gives three roots of c^2. Corresponding to these roots, there exist three longitudinal waves.

Transverse waves

Similarly, putting equation (8) into (7) we get

$$\left.\begin{aligned} (a_1x^2 + a_2)v_0^s + a_3v_0^f + a_4v_0^g &= 0 \\ b_1v_0^s + (b_2x^2 + b_3)v_0^f + b_4v_0^g &= 0 \\ d_1v_0^s + d_2v_0^f + d_3v_0^g &= 0 \end{aligned}\right\} \tag{13}$$

where,

$a_1 = \frac{\mu}{\rho^s}$; $a_2 = \frac{-(B^{ff} + B^{gg} + 2B^{fg})i}{\omega\rho^s} - 1$

$a_3 = \frac{(B^{fg} + B^{ff})i}{\omega\rho^s}$; $a_4 = \frac{(B^{fg} + B^{gg})i}{\omega\rho^s}$

$b_1 = \frac{-(B^{fg} + B^{ff})i}{\omega\rho^f}$; $b_2 = \frac{\beta i\omega}{\rho^f}$

$b_3 = \frac{B^{fg} i}{\omega\rho^f} + 1$; $b_4 = \frac{B^{fg} i}{\omega\rho^f}$

$d_1 = \frac{-(B^{fg} + B^{gg})i}{\omega\rho^g}$; $d_2 = \frac{B^{fg} i}{\omega\rho^g}$

$d_3 = \frac{B^{gg} i}{\omega\rho^g} + 1$

We can write the equation (13) in a matrix form as follows:

$$\begin{pmatrix} a_1x^2 + a_2 & a_3 & a_4 \\ b_1 & b_2x^2 + b_3 & b_4 \\ d_1 & d_2 & d_3 \end{pmatrix} \begin{pmatrix} v^s \\ v^f \\ v^g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

For the non trivial solution

$$det \begin{pmatrix} a_1x^2 + a_2 & a_3 & a_4 \\ b_1 & b_2x^2 + b_3 & b_4 \\ d_1 & d_2 & d_3 \end{pmatrix} = 0$$

Similarly, employing $x^2 = c^{-2}$ and solving the determinant we get,

$$dc^4 + bc^2 + a = 0 \tag{14}$$

where, $a = a_1b_2d_3$; $b = a_1b_3d_3 - a_1b_4d_2 + a_2b_2d_3 - a_4b_2d_1$;

$c = a_4b_1d_2 - a_3b_1d_3 + a_2b_3d_3 - a_2b_4d_2 + a_3b_4d_1 - a_4b_3d_1$.

Equation (14) is quadratic in c^2 and the solution of equation (14) gives two roots of c^2.

Corresponding to these roots, there exist two transverse waves. Therefore we conclude that there are two transverse waves.

PHASE VELOCITIES AND ATTENUATIONS

Phase velocities

In this section we shall find the explicit values of phase velocities (for longitudinal and transverse wave). Solving equation (12) and (15) we can find the values of velocities of longitudinal and transverse waves respectively.
Now solving equation (12) in c^2, we shall suppose that the solutions are c_1^2, c_2^2 and c_3^2, given as:

$$\left.\begin{aligned}
c_1^2 &= \left((H^2+I^3)^{\frac{1}{2}}-H\right)^{\frac{1}{3}} - I\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{-\frac{1}{3}} - \frac{C}{3D} \\
c_2^2 &= I\left(2\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{\frac{1}{3}}\right)^{-1} - \frac{C}{3D} - \frac{1}{2}\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{\frac{1}{3}} + \frac{\sqrt{3}}{2}\left(I\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{-\frac{1}{3}}\right) + \frac{i}{2}\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{\frac{1}{3}} \\
c_3^2 &= I\left(2\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{\frac{1}{3}}\right)^{-1} - \frac{C}{3D} - \frac{1}{2}\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{\frac{1}{3}} - \frac{\sqrt{3}}{2}\left(I\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{\frac{1}{3}}\right) + \frac{i}{2}I\left((H^2+I^3)^{\frac{1}{2}}-H\right)^{-\frac{1}{3}}
\end{aligned}\right\} \quad (15)$$

where, $H = \frac{A}{2D} + \frac{C^3}{27D^3} - \frac{BC}{6D^2}$ and $I = \frac{B}{3D} - \frac{C^2}{9D^2}$
And solving (14) we also have

$$\left.\begin{aligned}
c_4^2 &= \frac{-b+\sqrt{b^2-4ad}}{2d} \\
c_5^2 &= \frac{-b-\sqrt{b^2-4ad}}{2d}
\end{aligned}\right\} \quad (16)$$

The equation (15) and (16) gives the explicit values of longitudinal and transverse waves respectively. We may call coupled longitudinal waves as L1, L2, L3 waves respectively having phase velocities c_1^2, c_2^2, c_3^2. And let the couple shear waves be named them as S1, S2 waves respectively having phase velocities c_4^2, c_5^2.

Attenuations

Due to the involvement of Complex numbers in the coefficients in equations (12) and (14), then the solutions will also be in complex valued.[11] Therefore equations (15) and (16) gives complex velocities, so that the corresponding propagations of elastic waves will be attenuated. Let us denote the attenuations by the symbol p. The attenuation coefficients of the longitudinal and transverse waves are given by[12]

$$p_i = \left|\frac{\omega}{Im(c_i)}\right| \quad (17)$$

Where, $Im(c_i)$ denote the imaginary part of complex value of (c_i), and $i = 1, 2, 3, 4, 5$. The first three attenuations corresponding to longitudinal waves and last two correspond to transverse wave.

PARTICULAR CASES

If we neglect fluid and gas constituents then the equation (4) will become classical field equation. Further, consider that the parameters α, β and B^{fg} are tending to zero. Then the field equation (4) will be become classical field equation for poroelastic medium found by Tuncay and Corapcioglu. The coefficient B^{fg} is originated from the interactions of fluid and gas. And the coefficients α and β are coming from the macroscopic nature of viscous fluid. The existence of second shear wave is due to the

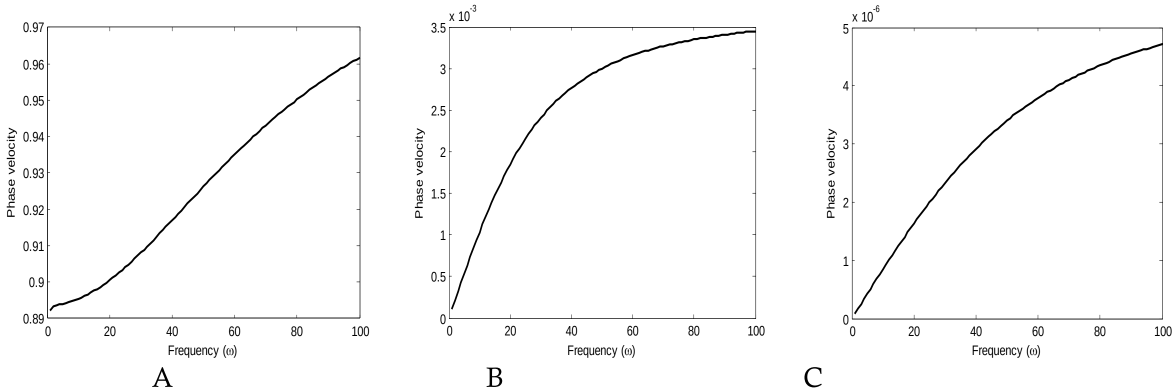


Figure 1. Variation of longitudinal wave velocities.

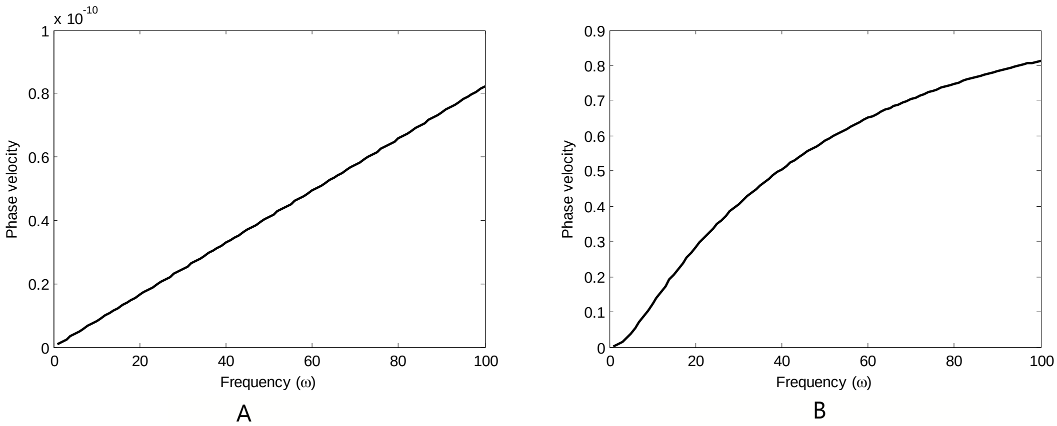


Figure 2. Variation of transverse wave velocities.

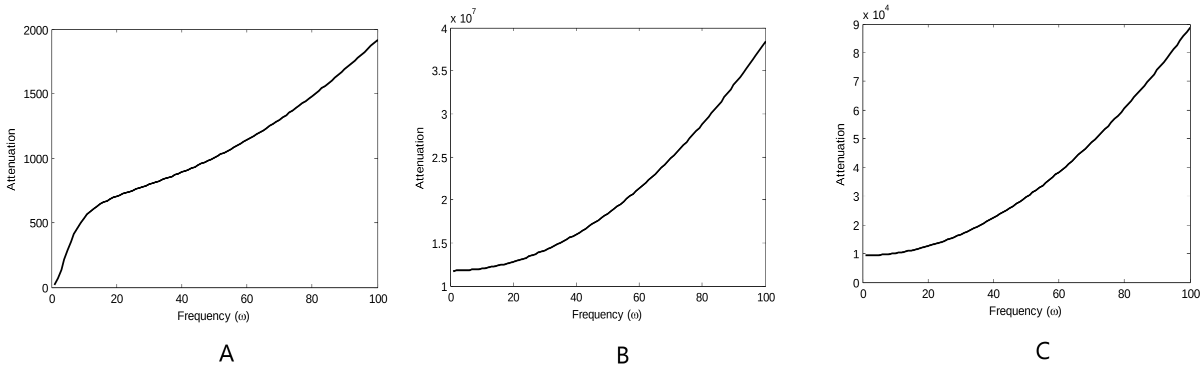
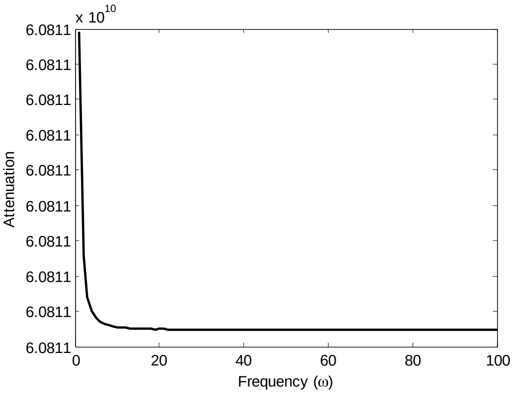
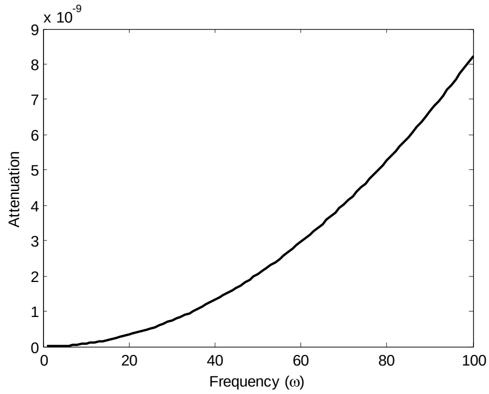


Figure 3. Variation of attenuation of longitudinal waves.



A



B

Figure 4. Variation of attenuation of transverse waves.

presence of β in the expression of t_{ij}^{f} . S2 is vanished in the absence of β.

If we neglect α, β, and B^{fg}, the medium will become classical one, and the expression of phase velocity of longitudinal waves will remain same, except the new definition of $E_1 = \frac{A^{ff}}{\rho^f}$. But the transverse S1 waves will be the only wave with the velocity ($c_4^{'}$) given by (see eq. 14)

$$c_4^{'2} = -\frac{b'}{d'} \quad (18)$$

where, $b^{'} = a_1 b_3 d_3$; $d^{'} = -a_4 b_3 d_1 - a_3 d_3 b_1 + a_2 b_3 d_3$

; ; ; ; .And attenuation, say $p_4^{'}$, is given by

$$p_4^{'} = \left|\frac{\omega}{Im(c_4)}\right| \quad (19)$$

Numerical Results and Computation

In this section we shall try to know the nature of phase velocities of dilatational and shear waves with respect to angular frequency. We shall use the following appropriate values of Elastic parameters, swelling parameters, fluid and gas parameters as follows:

$\lambda = 2.2238 \times 10^{10}\ N\,m^{-2}$; $\mu = 2.992 \times 10^{10} N\,m^{-2}$;
$\alpha = 1.002 \times 10^{-3} N\,sec\,m^{-2}$; $\beta = 8.88 \times 104\ N\,sec\,m^{-2}$;
$A^f = -1.3 \times 10^6 N\,m^{-2}$; $A^{ff} = -3.7 \times 10^5 N\,m^{-2}$; $A^{fg} = -2.45 \times 10^5\ N\,m^{-2}$;
$A^f = -1.2 \times 10^4 N\,m^{-2}$; $A^{gg} = -1.7 \times 10^5 N\,m^{-2}$;
$B^{ff} = 4 \times 10^6 N\,sec\,m^{-4}$; $B^{fg} = 5\ N\,sec\,m^{-4}$; $B^{gg} = 3{:}3 \times 10^4\ N\,sec\,m^{-4}$;
$\rho^s = 5 \times 10^5\ Kg\,m^{-3}$; $\rho^f = 6 \times 10^4 Kg\,m^{-3}$; $\rho^g = 1 \times 10^3 Kg\,m^{-3}$

To illustrate the nature of phase velocities and its attenuation dependence on angular frequency we shall use the above given values of parameters in to the equations (15-19), to compute them numerically. And the dependence of phase velocities and its attenuations with respect to angular frequency has been depicted graphically.

Figure 1 (A, B, C) showed variation of Phase velocities of longitudinal waves with respect to angular frequency. We see that the phase velocities are increases as increasing frequency. The first velocity (c_1^2) of L1 wave is fastest and third velocity (c_3^2) of L3 wave is slowest among the three longitudinal waves.

The Figure 2 (A, B) show variation of Phase velocities of transverse waves with respect to angular frequency. We see that the phase velocities are increases as increasing frequency. The velocity (c_4^2) of S1 wave is very slow as compared to the velocity (c_5^2) of S2 wave.

We knew that the five waves were attenuating in nature. The following Figures 3 (A, B, C) show the variation of attenuations of longitudinal waves with respect to angular frequency. The values of attenuations are increases as increasing angular frequency.

And the following Figure 4 (A, B) show variations of transverse waves. In fig 4 (A), we see that the transverse wave (S1) is very slow and highly attenuated. And from fig 4 (B), we see that S2 waves has lower attenuation as compare to S1 waves.

Conclusion

In homogeneous isotropic swelling porous materials (containing solid, liquid and gas) there are five waves – three coupled longitudinal and two coupled transverse waves propagating with phase speed c_i^2, (i=1, 2, 3, 4, 5), they are all frequency dependent and dispersive. The propagation of wave in this medium is attenuating in nature. The existence of S2 wave is due to the presence of coefficient of viscosity.

References

1. Biot MA (1956). The theory of elastic waves in a fluid-saturated porous solid. *J Acaust Soc Am*, **28**, 168–191.
2. Bowen RM (1980). Incompressible porous media models by use of the theory of mixtures. *Int J Engng Sci,* **18**, 1129–1148.
3. Grag SK & Nayfeh AH (1986). Compressional wave propagation in liquid and gas saturated elastic porous media. *J Appl Phys*, **60**, 3045–3055.
4. Tuncay K & Corapcioglu MY (1997). Wave propagation in poroelastic media saturated by two fluids. *J Appl Mech*, **64**, 313–320.
5. Gales C (2004). Waves and vibrations in the theory of swelling porous elastic soils. *Eur J Mech A*, **23**, 345–357.
6. Kumar R, Divya & Kumar K (2013). Wave propagation in swelling porous thermoelastic half space. *Int J Appl Math Mech,* **9**, 1–30.
7. Tersa W & Bennethum S (2006). On the derivation of the transport equation for swelling porous materials with _finite deformation. *Int J Engng Sci*, **44**, 1408–1442.
8. Gales C (2002). Some uniqueness and continuous dependence results in the theory of swelling porous elastic soils. *Int J Engng Sci*, **40**, 1211–1231.
9. 9. Boffil F& Quintanilla R (2003). Anti-plane shear deformation of swelling porous elastic soils. *Int J Engng Sci,* **41**, 801–816.
10. Eringen AC (1994). A continuum theory of swelling porous elastic soils. *Int J Engng Sci*, **32**, 1337–1349.
11. Tomar SK & Goyal S (2013). Elastic waves in swelling porous media. *Transp Porous Med*, **100**, 39–68.
12. Kumar R & Kansal T (2012). Propagation of plane waves and fundamental solution in the theories of thermoelastic diffusive materials with voids. *Int J Appl Math Mech,* **8**, 84–103.