



Application of mathematics in business appraisal

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ABSTRACT

Important mathematical tools/techniques for appraisal of business projects under certainty, risk and uncertainty conditions are discussed in this paper, including certainty equivalent method, risk adjusted discount rate method, expected monetary value (EMV) method, expected opportunity loss (EOL) method, criterion of optimism, criterion of pessimism and criterion of realism. Under Certainty equivalent and risk adjusted discount rate methods net present value (NPV) of the project/investment project is found out to enable the decision maker or the project manager whether to continue or reject or select the best among the alternatives. In cases of the business decision making under risk or probabilistic decision situations EMV and EOL are suitably used. In the absence of knowledge about the probability of any state of nature (future) occurring, the decision maker must base his decision only on the actual conditional pay off values. In such situations criterion of optimism, criterion of pessimism and criterion of realism as Hurwicz criterion are usefully applied.

Key words: Mathematical methods; business appraisal; NPV; EMV; EOL; maximax; maximin.

INTRODUCTION

Mathematical methods/tools are used for various purposes in various fields of study. There are various methods of mathematics which are useful for the analysis, appraisal and in the decision making processes of business,^{1,2} economics³ and commerce.⁴⁻⁶ When we have/known some expected values, of the investment proposals with certainty equivalent coefficient values, certainty equivalent method can be used to transform expected uncertain flows into certainty equivalent. Un-

der this method net present value (NPV) of the project/investment project is found out to enable the decision maker or the project manager whether to continue or reject or select the best among the alternatives.

The NPV is mathematically calculated as per formula

$$NPV = \sum_{t=1}^n \frac{\alpha_t \bar{A}_t}{(1+i)^t} - I$$

where NPV = net present value, \bar{A}_t = expected cash flow for the year 't', α_t = certainty equivalent coefficient of the cash flow of year 't', i = risk free interest rate and I = initial invest-

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ment. Similarly risk adjusted discount rate is also used in such situations. Mathematically

$$NPV = \sum_{t=1}^n \frac{\bar{A}_t}{(1+r_k)^t} - I$$

where r_k = risk adjusted discount rate for the project 'k'.

In cases of the business decision making under risk or probabilistic decision situations, EMV and EOL are most widely used.⁷ Mathematically EVM IS

$$EMV(S_j) = \sum_{i=1}^m P_{ij}P_i$$

where, $EMV(S_j)$ = EMV of the course of action S_j , P_{ij} = Payoff associated with state of nature N_i and course of action S_j , P_i = Probability of occurrence of the state of nature 'i' or N_i . EOL is stated mathematically as

$$EOL(N_i) = \sum_{j=1}^m l_{ij}P_j$$

where $EOL(N_i)$ = EOL of the state of nature N_i , l_{ij} = Opportunity loss due to state of nature N_i and course of action S_j , P_j = Probability of occurrence of the state of nature N_j . In the absence of knowledge about the probability of any state of nature (future) occurring, the decision maker must base his decision only on the actual conditional pay off values, in which criterion of optimism, criterion of pessimism and criterion of realism as Hurwicz criterion are applied.⁷

APPLICATION OF CERTAINTY EQUIVALENT METHOD

This method finds out the net present

value (NPV) of the investment proposal with the help of certainty equivalent coefficient. It transforms expected values of uncertain cash flows into their certainty equivalents. The risk free rate is applied for discounting purposes. It is also assumed that there is no uncertainty in regard to initial investment/ investment outlay money of the project.

Under certainty equivalent method the NPV is calculated by

$$NPV = \sum_{t=1}^n \frac{\alpha_t \bar{A}_t}{(1+i)^t} - I$$

Where, NPV= net present value, \bar{A}_t = expected cash flow for the year 't', α_t = certainty equivalent coefficient of the cash flow of year 't', i = risk free interest rate and I = initial investment.

The value of the certainty equivalent coefficient (CEC) usually ranges between 0.5 and 1.0. A value of 1 implies that the cash flow is certain or the management is risk neutral. In industrial situations however, cash flows are generally uncertain and managements usually risk adverse. Hence the CEC is typically less than 1. The CEC is not same for different types of investments and a relevant coefficient is applied base on the type of investment.⁸ Table 1 shows the CEC for different types of investments

This method can be applied in the following business situation:

APPLICATION 1

XY company is considering an investment proposal for expansion of its business. The

Table 1. Certainty Equivalent Coefficient (CEC).

SI No	Type of investment	Certainty equivalent coefficient			
		Year 1	Year 2	Year 3	Year 4
1.	Replacement investment	0.92	0.87	0.84	0.80
2.	Expansion investments	0.89	0.85	0.80	0.75
3.	New product investment	0.85	0.80	0.74	0.68
4.	Research & Development investments	0.75	0.70	0.64	0.58

investment outlay will be Rs. 5,00,000. Expected cash flow from the project for 4 years and CEC are as follows:

Year	Expected cash flow	CEC
1	Rs. 1,50,000	0.89
2	Rs. 1,70,000	0.85
3	Rs. 2,00,000	0.80
4	Rs. 2,50,000	0.75

The risk free rate of interest is 5 percent.

From these information we can find out whether the project will be profitable i.e required to expand or not profitable i.e. not advisable for expansion.

Calculation of NPV:

$$\begin{aligned}
 NPV &= \sum_{t=1}^n \frac{\alpha_t \bar{A}_t}{(1+i)^t} - I \\
 &= \frac{1,50,000 \times (0.89)}{1.05} + \frac{1,70,000 \times (0.85)}{(1.05)^2} \\
 &\quad + \frac{2,00,000 \times (0.80)}{(1.05)^3} + \frac{2,50,000 \times (0.75)}{(1.05)^4} \\
 &\quad - 5,00,000 \\
 &= \frac{1,33,500}{1.05} + \frac{1,44,500}{1.1025} + \frac{1,60,000}{1.1576} \\
 &\quad + \frac{1,87,500}{1.2155} - 5,00,000.
 \end{aligned}$$

$$= 5,50,683 - 5,00,000 = 50,683$$

As the NPV is positive and reasonable amount, the management can take decision for expansion of the business.

Certainty equivalent method is conceptually superior to the risk adjusted discount rate method because it does not assume that risk increase with time at a certain rate. Each year's CEC is based on the level of risk characterizing its cash flow. Despite its conceptual soundness it is not as popular as the risk adjusted discount rate method. This is perhaps because it is inconvenient and difficult to specify a series of certainty equivalent coefficients but seemingly simply to adjust the discount rate.^{8,9}

APPLICATION OF RISK ADJUSTED DISCOUNT RATE (RADR) METHOD

The RADR method calls for adjusting the discount rate to reflect project risk. If the risk of a project is equal to the existing investments of the firm, the discount rate used is the average cost of capital of the firm; if the risk of the project is greater than the risk of the existing investment of the firm, the discount rate used is higher than the average cost of capital of the firm. If the risk of the project is less than the risk of the existing investment of the firm, the rate used is less than of the average cost of capital of the firm.⁸

The risk adjusted discount rate is stated mathematically as:

$$r_k = i + n + d_k$$

Where, r_k = risk adjusted discount rate for the project k; i = risk free rate of interest, n = adjustment for the firms normal risk, d_k = adjustment for the differential risk of project k.

It may be noted that $(i+n)$ measures the firms cost of capital, d_k may be positive or negative depending on how the risk of project under consideration compares with the existing risk of the firm.

The adjustment for differential risk of project k, quite understandably, depends on the management's perception of the project risk and management's attitude towards risk (risk – return preference). A large pharmaceutical concern, (as an example) uses the following risk adjusted discount rate for various types of investments:-

Table 2. Risk adjusted discount rate.

Sl. No.	Investment category	Risk adjusted discount Rate
1.	Replacement investment	Cost of capital
2.	Expansion investment	Cost of capital + 3%
3.	Investment in rerated lines	Cost of sapital +6%
4.	Investment in new lines	Cost of capital +10%

Once the project risk adjusted discount rate (r_k) is specified, the project is accepted if its net profit value (NPV), calculated is positive. NPV is calculated by

$$NPV = \sum_{t=1}^n \frac{\bar{A}_t}{(1+r_k)^t} - I$$

APPLICATION 2

M/S X & Co. is considering a project for investment in a new product line which will involve an investment outlay of Rs. 2,00,000. The expected cash flows from the project for the period of 5 years are as follows:

Year	Cash flow (Rs)
1	40,000
2	55,000
3	45,000
4	50,000
5	42,000

The risk adjusted discount rate is 18%.

From this information the management / decision maker can take a decision that, whether this project is worthwhile or not.

Calculation of NPV using the RADR method:

$$\begin{aligned}
 NPV &= \sum_{t=1}^n \frac{\bar{A}_t}{(1+r_k)^t} - I \\
 &= \frac{40,000}{1.18} + \frac{55,000}{(1.18)^2} + \frac{45,000}{(1.18)^3} + \frac{50,000}{(1.18)^4} \\
 &\quad + \frac{52,000}{(1.18)^5} - 2,00,000 \\
 &= 33,898 + 76,560 + 27,557 + 25,785 + 18,341 - 2,00,000 \\
 &= 1,82,141 - 2,00,000 = - 17,859
 \end{aligned}$$

Discussion

Since the NPV is negative the project is not worthwhile. Therefore, it is advisable that the management can take the decision not to continue the project.

The RADR is commonly employed in

practice. Firm use different discount rates, presumably related to the risk factor, for different types of investment project. The discount rate is generally low for routine replacement investments, moderate for expansion and high for new investments. Despite its popularity, RADR method suffers from two serious limitations:

1. It is difficult to estimate d_k consistently.
2. This method assumes that risk increases with time at a constant rate. This assumption may not be valid.

DECISION MAKING UNDER RISK

Decision making under risk is a probabilistic decision situation in which more than one state of nature exists and the decision maker has sufficient information to assign probability values to the likely occurrence of each of these states ¹⁰. Knowing the probability distribution of the states of nature, the best decision is to be selected that course of action which has the largest expected payoff value. The most widely used mathematical techniques for various course of action under risk is the expected monetary value (EMV) and expected opportunity loss (EOL) methods ¹¹.

APPLICATION OF EMV

The EMV for a given course of action is the weighted average payoff which is the sum of the payoffs for each course of action multiplied by probabilities associated with each state of nature.

Mathematically EMV is stated as follows:

$$EMV(S_j) = \sum_{i=1}^m P_{ij}P_i$$

Where, $EMV(S_j)$ = EMV of the course of action S_j , P_{ij} = payoff associated with state of nature N_i and course of action S_j , P_i = probability of occurrence of the state of nature 'i' or N_i .

Steps in Calculating EMV

1. a) Construct a payoff matrix listing all possible courses of action and states of nature. b) Enter the Conditional payoff values associated with each possible combination of course of action and state of nature along with the probabilities of the occurrence of each state of nature.

2. a) Calculate the EMV for each course of action by multiplying the conditional payoffs by the associated probabilities; and b) Add these weighted values for each course of action.

3. Select the course of action that yields, the optimal EMV.

APPLICATION 3

Mr. Y quite often flies from Aizawl to Kolkata. He can use the airport bus which costs Rs. 100 but if he takes it, there is a 0.08 chance that he will miss the flight. A hotel AZ costs Rs. 650 with a 0.96 chance of being on time for the flight. For Rs. 750 he can use a taxi which will make chances of getting 99 of 100 flights. If Mr. Y catches the flight on time, he will conclude a business transaction which will produce a profit of Rs. 25,000, otherwise he will lose it. Here, which mode of transport should Mr. Y use? This can be answered by using the EMV criterion.

Now, let us compute the EMV for this business situation (Table 3).

Comparing the EMV associated with each course of action indicates that course of action 'taxi' is the logical alternative because it

Table 3. Computation of EMV of various course of action.

State of Nature	Courses of action								
	Airport bus			Hotel			Taxi		
	Cost	Probability	E.V.	Cost	Probability	E.V.	Cost	Probability	E.V.
Catch the flight	25,000-100 =24,900	0.92	22,908	25,000-650 =24,350	0.96	23,376	25,000-750 = 24,250	0.99	24,007.5
Miss the flight	- 100	0.08	-8.0		0.04	-6.0	-750	0.01	-7.5
EMV		22,900			23,350			24,000	

has the highest EMV.

APPLICATION 4

M/s. Z flower shop is delivering flower to its customers daily in the morning. This flower promises its customers delivery within three hours on all flowers orders. All flowers are purchased on the previous day and delivered by the next morning. Based on the past experience it is found that, the demand for roses in a particular day range from 70 bundles of flowers (minimum) to 100 bundles of flowers (maximum). The shop purchases in minimum margin of 10 bundles. Some of the information regarding daily demand for roses with probability is given below:

Demand in bundle of roses	Probability
70	0.1
80	0.2
90	0.4
100	0.3

M/s Z purchases roses for Rs. 30 per bundle of roses and sells them for Rs. 50 per bundle. All unsold roses are donated to a local Hospital. In this business situation, How many bundle of flowers should M/s Z purchase to maximize profit. This can be decided by using EMV method.

From the above business situation we had to find out the "course of action" (decision choice) and "state of nature" (event).

Since the number of bundle of roses purchase is under the control of decision maker, purchase per day is considered as "course of action"(decision choice) and daily demand of

the flower is uncertain and only known with probability, therefore it is consider as “state of nature” (event).

From the data we clear that the flower shop must not purchase less than 70 bundles or not more than 100 bundles of roses per day. Each bundle of roses sold within a day yields a profit of Rs. (50-30) =20 and otherwise it is a dead lose of Rs. 30.

Thus marginal profit (MP) = selling price – cost = Rs. (50 – 30) = Rs. 20. Marginal loss (ML) = loss on unsold roses =Rs. 30

Using the information given in the information, the various conditional profit (Payoff) values for each combination of decision choice – event is given by:

Conditional Profit = MP x roses sold – ML x roses not sold = 20 D, if D is >= S; Or 20D – 30 (S-D) = 50D – 30S, if D is < S. Where, D denotes the number of roses sold within a day and S the number of roses stock (purchase).

Computation of conditional profit values and corresponding expected payoffs:-

Table 4. Conditional profit values (payoffs).

State of Nature (Demand per day)	Probability (1)	Conditional payoff				Expected payoff			
		Course of action (Purchase per day)				Course of action (purchase per day)			
		(2)				(1) x (2)			
		70	80	90	100	70	80	90	100
70	0.1	1400	1100	800	500	140	110	80	50
80	0.2	1400	1600	1300	1000	280	320	260	200
90	0.4	1400	1600	1800	1500	560	640	720	600
100	0.3	1400	1600	1800	2000	420	480	540	600
EMV						1400	1550	1600	1450

Since the highest EMV of Rs.1600, is corresponding to course of action 90, The M/s Z flower shop should purchase ninety (90) bundles of roses every day to maximize its profit.

APPLICATION OF EOL

An alternative approach to maximizing

expected monetary value (EMV) is to minimize the expected opportunity loss (EOL). The EOL is defined as the difference between the highest profit (or payoffs) for the particular course of action taken. The course of action due to which EOL is minimum is recommended.

Since EOL is an alternative decision criterion for decision making under risk, therefore the result will always be the same as those obtained by EMV criterion. Thus only one of the two methods will be applied to reach a decision.

Mathematically EOL is stated as:

$$EOL(N_i) = \sum_{i=1}^m l_{ij}P_i$$

Where, EOL(N_i) = EOL of the state of nature N_i, l_{ij} = opportunity loss due to state of nature N_i and course of action S_j, P_i = probability of occurrence of the state of nature N_i.

Steps for calculating EOL

1. Prepare a conditional payoff table for each course of action- state of nature along with the associated probabilities.
2. For each state of nature calculate the conditional opportunity loss (COL) values by subtracting each payoff from

- the maximum payoff for the year.
3. Calculate EOL for each course of action by multiplying the probability of each state of nature with the COL value and then adding the values.
 4. Select a course of action for which the EOL value is minimum.

APPLICATION 5

A company manufactures goods for a market in which the technology of the product is changing rapidly. The research & development has produced a new product which appears to have potential for commercial exploitation. A further Rs. 80,000 is required for development testing.

The company has 100 customers and each customer might purchase at the most one unit of the product. Market research suggests that a selling price of Rs. 8,000 for each unit with total variable costs of manufacture and selling estimate are Rs. 2,500 for each unit.

From the various experience, it has been possible to drive a probability distribution relating to the proportions of customers who will buy the product as follows:-

Proportion of customers:	0.04	0.08	0.12	0.16	0.20
Probability:	0.1	0.1	0.2	0.4	0.2

From the above information by determining EOL the company can take decision that whether or not they should develop the product.

If 'P' is the proportion of customers who purchase the new product, then the conditional profit is:

$$(8,000 - 2,500) \times 100 p - 80,000 = \text{Rs. } (5,50,000 - 80,000)$$

Let 'N_i' (i=1,2,3,4 & 5) be the possible state of nature, i.e. proportion of the customers who will buy the new product; and S₁ (develop the product) and S₂ (do not develop the product) to be the two courses of actions.

Calculation of conditional profit values (payoffs) for each pair of N_i's and S_j's:

Table 5. Conditional payoffs.

State of nature (Proportion of customers, P)	Conditional profit = Rs.(550000p - 80000)	
	Course of action S ₁	Course of action S ₂
0.04	(550000x0.04- 80000) = - 58,000	0
0.08	(550000x0.08- 80000) = - 36,000	0
0.12	(550000x0.12- 80000) = - 14,000	0
0.16	(550000x0.16- 80000) = 8000	0
0.20	(550000x0.20- 80000) = 30000	0

Calculation of opportunity loss values is given in Table 6.

CALCULATION OF EOL

Using the given estimates of probabilities associated with each state of nature, the expected opportunity loss (EOL) for each course of action is given below:

$$\text{EOL (S}_i) = (0.1) (58,000) + 0.1 (36,000) + 0.2 (14,000) + 0.4 (0) + 0.2 (0)$$

Table 6. Opportunity loss.

State of nature (Proportion of customers, P)	Probability	Conditional profit (Rs.)		Opportunity loss (Rs.)	
		Course of action S ₁	Course of action S ₂	Course of action S ₁	Course of action S ₂
0.04	0.1	- 58,000	0	58,000	0
0.08	0.1	- 36,000	0	36,000	0
0.12	0.2	- 14,000	0	14,000	0
0.16	0.4	8000	0	0	8000
0.20	0.2	30000	0	0	30000

$$\begin{aligned} &= 5,800 + 3,600 + 2,800 + 0 + 0 = 12,200. \\ \text{EOL } (S_2) &= (0.1) (0) + 0.1 (0) + 0.2 (0) + \\ &0.4 (8,000) + 0.2 (30,000) \\ &= 0 + 0 + 0 + 3,200 + 6,000 = 9,200. \end{aligned}$$

Since the company seeks to minimize the expected opportunity loss, the company should select the course of action S_2 (Do not develop the product) with minimum EOL.

DECISION MAKING UNDER UNCERTAINTY

In the absence of knowledge about the probability of any state of nature (future) occurring, the decision maker must base his decision only on the actual conditional payoff values, together with a policy (attitude). There are several different criteria of decision making in this situation. Criterion of optimism, criterion of pessimism and criterion of realism (Hurwicz criterion) are some of such important methods/criteria.¹²⁻¹⁴

(i) Application of criterion of optimism (Maximax and Minimin)

In this criterion the decision maker ensures that he should not miss the opportunity to achieve the greatest possible payoff or the lowest cost. Thus, he selects the alternative decision choice that represents the maximum of the maxima payoff or minimum of the minima (in case of loss or cost). The working method is summarized as follows:

1. Locate the maximum payoff values or minimum (in case of cost) corresponding to each alternative (or course of action), then
2. Select an alternative with best anticipated payoff values (maximum for profit and minimum for cost).

(ii) Application of criterion of pessimism (Minimax or maximin)

In this criterion the decision maker ensures

that he should earn no less (or pay no more) than some specified amount. Thus he select the alternative that represents the maximum of the minima payoffs (in case of profit) or minimum of maxima (in case of cost). The working method is summarized as following:

1. Locate the minimum (in case of cost or loss) or maximum (in case of profit) pay off values corresponding to each alternative, then
2. Select an alternative with the best anticipated payoff value (maximum for profit and minimum for cost or loss).

(iii) Application of criterion of realism (Hurwicz criterion)

This criterion suggests that a rational decision-maker should be neither completely optimistic nor pessimistic, and therefore must display a mixture of both. Hurwicz, who suggested this criterion, introduced the idea of a coefficient of optimism (denoted by α) to measure the decision-maker's degree of optimism. This coefficient lies between '0' and '1', where '0' represents a completely pessimistic attitude about the future and '1' a completely optimistic attitude about the future. Thus, if ' α ' is the coefficient of optimism, then $(1-\alpha)$ will represent the coefficient of pessimism.

The Hurwicz approach suggest that the decision-maker must select an alternative that maximizes 'H' (Criterion Realism):

H (Criterion of Realism) = α x (maximum in Column) + $(1-\alpha)$ x (Minimum in Column).

The working method is summarized as follows:

1. Decide the coefficient of optimism ' α ' and then coefficient of pessimism $(1 - \alpha)$.
2. For each alternative select the largest and lowest payoff values and multiply with ' α ' and $(1-\alpha)$ values respectively. Then calculate the weighted average, 'H' by using the above formula.

3. Select an alternative with best anticipated weighted average payoff value.

APPLICATION 7

A food product company in contemplating the introduction of revolutionary new product with new packaging to replace the existing product at much higher price (S₁) or a moderate change in the composition of the existing product with a new packaging at a small increase in price (S₂) or a small change in the composition of the Existing, except the word 'New' with a negligible increase in price (S₃). The Three possible states of nature or event are: (i) high increase in Sales (N₁), (ii) No change in Sales (N₂) and (iii) increase in sales (N₃). The marketing Department of the company worked out the pay offs in terms of yearly net profits for each of the strategies of the three events (expected sales). This represented in the following table:

Table 7. Payoff matrix table.

Strategies	State of Nature		
	N ₁	N ₂	N ₃
S ₁	6,00,000	3,00,000	1,50,000
S ₂	5,00,000	4,50,000	0
S ₃	3,00,000	3,00,000	3,00,000

From this information the executive can choose the best alternative on the basis of Maximax, Maximin and Hurwicz criterion.

(i) Decision based on Maximax criterion:

Table 8. Re-written payoff matrix-1.

State of Nature	Strategies		
	S ₁	S ₂	S ₃
N ₁	6,00,000	5,00,000	3,00,000
N ₂	3,00,000	4,50,000	3,00,000
N ₃	1,50,000	0	3,00,000
Column maximum	6,00,000	5,00,000	3,00,000

The maximum of column maxima is 6,00,000. Here the executive of the company should take decision to adapt the strategy S₁ i.e. introduction of a new product to replace the existing product at much higher price.

(ii) Decision under Maximin criterion:

Table 9. Re-written payoff matrix-2.

State of Nature	Strategies		
	S ₁	S ₂	S ₃
N ₁	6,00,000	5,00,000	3,00,000
N ₂	3,00,000	4,50,000	3,00,000
N ₃	1,50,000	0	3,00,000
Column maximum	1,50,000	0	3,00,000

The maximum of column minima is 3,00,000. Hence the company should adapt the strategy S₃ i.e. a small change in the composition of the existing product with negligible increase in price.

(iii) Decision in regard to Hurwicz criterion:

Let 'α' be the coefficient of optimism and value α be 0.7.

Then value of coefficient of pessimism = (1- 0.7) = 0.3.

Table 10. Re-written payoff matrix-3.

State of Nature	Strategies		
	S ₁	S ₂	S ₃
N ₁	6,00,000	5,00,000	3,00,000
N ₂	3,00,000	4,50,000	3,00,000
N ₃	1,50,000	0	3,00,000
Column maximum	6,00,000	5,00,000	3,00,000
Column minimum	1,50,000	0	3,00,000
H =	0.7x6,00,000	0.7x5,00,000	0.7x3,00,000
α(Maximum in column)	+ 0.3x 1,50,000	0.3 x 0 = 3,50,000+ 0	+ 0.3x 3,00,000 = 2,10,000+
+	= 4,20,000+	= 3,50,000.	= 2,10,000+
(1-α) (Minimum in column)	45,000 = 4,65,000.		90,000 = 3,00,000.

Among the alternatives course of action S_1 has the highest/maximum value of H (criterion of realism), therefore, course of action/ strategy S_1 i.e. introduction of a new product to replace the existing product at much higher price will be selected.

CONCLUSION

From the above applications of mathematical methods/tools in the business decision making, we conclude that, mathematical methods are very important and helpful in business decision making processes in various business situations. Under the certainty business conditions where there is probable cash flows are expected, certainty equivalent method and risk adjusted discount rate method are very useful. Under certainty equivalent method, the expected cash flows of the project are converted into their certainty equivalents by applying suitable certainty equivalent coefficients. Then, the risk free rate is applied for discounting purposes. Under this method, the project having positive and higher NPV is selected among the alternatives. The risk adjusted discount rate method uses for adjusting discount rate to reflect the project risk. It uses a risk adjusted discount rate which is equal to the risk free rate plus adjustment for the firms normal risk and adjustment for the differential risk of the respective project, i.e., mathematically, $r_k = i + n + d_k$. While taking decision the project having positive and higher NPV is selected.

The EMV and EOL are useful for taking business decisions under probabilistic decision situations. Among the alternative courses of actions, the course of action which yields the optimal EMV is selected. That represents the course of action having highest monetary value. Under EOL, since it expects the minimum opportunity loss in the course of actions, selects the course of action which has minimum EOL value.

If the availability of information for a decision environment is incomplete, then the de-

cision taken under such an environment is called decision under uncertainty. Under such situation, the information of the decision environment cannot be described in the form of a probabilistic distribution. Under such situations criterion of optimism (Maximax and Minimin), criterion of pessimism (Minimax or maximin) and criterion of realism (Hurwicz criterion) are used. In the criterion of optimism (Maximax and Minimin) the decision maker ensures that he should not miss the opportunity to achieve the greatest possible payoff or the lowest cost. Thus, he selects the alternative decision choice that represents the maximum of the maxima payoff or minimum of the minima (in case of loss or cost).

In criterion of pessimism (Minimax or maximin) the decision maker ensures that he should earn no less (or pay no more) than some specified amount. Thus he select the alternative that represents the maximum of the minima payoffs (in case of profit) or minimum of maxima (in case of cost).

The Hurwicz criterion or criterion of realism which is denoted by "H" suggests that a rational decision-maker should be neither completely optimistic nor pessimistic, and therefore must display a mixture of both. Hurwicz introduced the idea of a coefficient of optimism to measure the decision-maker's degree of optimism. This coefficient lies between '0' and '1', where '0' represents a completely pessimistic attitude about the future and '1' a completely optimistic attitude about the future. While taking decision, among the alternative courses of actions the alternative which has highest/maximum value of "H" is selected.

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